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ECONOMIC MODEL FOR NEUTRINO MASSES AND MIXINGS

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Working in the framework of three chiral neutrinos with Majorana masses, we investigate a scenario where the neutrino mass matrix is strictly off-diagonal in the flavor basis, with all its diagonal entries precisely zero.

This talk is based on a publication with Glashow[1] to which I refer for more details than can be accommodated in this write-up. The minimal standard model involves three chiral neutrino states, but it does not admit renormalizable interactions that can generate neutrino masses. Nevertheless, experimental evidence suggests that both solar and atmospheric neutrinos display flavor oscillations, and hence that neutrinos do have mass. Two very different neutrino squared-mass differences are required to fit the data:

$$10^{-11}\mathrm{eV}^2 \le \Delta_s \le 10^{-5}\mathrm{eV}^2 \quad \text{and} \quad \Delta_a \simeq 10^{-3}\mathrm{eV}^2 \tag{1}$$

where the neutrino masses m_i are ordered such that:

$$\Delta_s \equiv |m_2^2 - m_1^2| \quad \text{and} \quad \Delta_a \equiv |m_3^2 - m_2^2| \simeq |m_3^2 - m_1^2|$$

and the subscripts s and a pertain to solar and atmospheric oscillations respectively. The large uncertainty in Δ_s reflects the several potential explanations of the observed solar neutrino flux: in terms of vacuum oscillations or large-angle or small-angle MSW solutions, but in every case the two independent squared-mass differences must be widely spaced with

$$r \equiv \Delta_s/\Delta_a < 10^{-2}$$

Solar neutrinos may exhibit an energy-independent time-averaged suppression due to Δ_a, as well as energy-dependent oscillations depending on Δ_s/E. Atmospheric neutrinos may exhibit oscillations due to Δ_a, but they are almost entirely unaffected by Δ_s. It is convenient to define neutrino mixing angles as follows:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_2c_3 & c_2s_3 & s_2e^{-i\delta} \\ +c_1s_3 + s_1s_2c_3e^{i\delta} & -c_1c_3 - s_1s_2s_3e^{i\delta} & -s_1c_2 \\ +s_1s_3 - c_1s_2c_3e^{i\delta} & -s_1c_3 - c_1s_2s_3e^{i\delta} & +c_1c_2 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

with s_i and c_i standing for sines and cosines of θ_i. For neutrino masses satisfying Eq.(1) the vacuum survival probability of solar neutrinos is[2]

$$P(\nu_e \to \nu_e)\Big|_s \simeq 1 - \frac{\sin^2 2\theta_2}{2} - \cos^4\theta_2 \sin^2 2\theta_3 sin^2(\Delta_s R_s/4E) \tag{2}$$

whereas the transition probabilities of atmospheric neutrinos are:

$$\begin{array}{rcl} P(\nu_\mu \to \nu_\tau)\Big|_a & \simeq & \sin^2 2\theta_1 \cos^4\theta_2 \ \sin^2(\Delta_a R_a/4E) \\ P(\nu_e \leftrightarrow \nu_\mu)\Big|_a & \simeq & \sin^2 2\theta_2 \sin^2\theta_1 \ \sin^2(\Delta_a R_a/4E) \\ P(\nu_e \to \nu_\tau)\Big|_a & \simeq & \sin^2 2\theta_2 \cos^2\theta_1 \ \sin^2(\Delta_a R_a/4E) \end{array} \tag{3}$$

None of these probabilities depend on δ, the measure of CP violation in the lepton sector.

Let us turn to the origin of neutrino masses. Among the many renormalizable and gauge-invariant extensions of the standard model that can do the trick is the introduction of a charged singlet meson f^+ coupled antisymmetrically to pairs of lepton doublets *and* (also antisymmetrically) to a pair of Higgs doublets. This simple mechanism was first proposed by Zee[3] and results (at one loop) in a Majorana mass matrix in the flavor basis (e, μ, τ) of a special form:

$$\mathcal{M} = \begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & 0 & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & 0 \end{pmatrix} \tag{4}$$

Related discussions of Eq.(4) appear elsewhere[4,5] The present work is essentially a continuation of [4].

The sum of the neutrino masses (the eigenvalues of $\mathcal{M}$) vanishes:

$$m_1 + m_2 + m_3 = 0 \tag{5}$$

An important result emerges when the squared-mass hierarchy Eqs.(1) is taken into account along with Eq.(5). In the limit $r \to 0$, two of the squared masses must be equal. There are two possibilities. In case A, we have $m_1 + m_2 = 0$ and $m_3 = 0$. This case arises iff at least one of the three parameters in $\mathcal{M}$ vanishes. In case B, we have $m_1 = m_2$ and $m_3 = -2m_2 > 0$. This case arises iff the three parameters in $\mathcal{M}$ are equal to one another. Of course, r is small but it does not vanish: in neither case can the above relations among neutrino masses be strictly satisfied. But they must be nearly satisfied. Consequently we may deduce certain approximate but useful restrictions on the permissable values of the neutrino mixing angles θ_i. Prior to examining these restrictions, we note that Eqs.(5) and (1) exclude the possibility that the three neutrinos are nearly degenerate in mass.

In [1] these cases are analysed carefully and the best version is an example of Case A with $m_{\mu\tau} = 0$ and leads to conservation of $L_e - L_\mu - L_\tau$. For this subcase, we obtain $\sin\theta_2 = 0$ and $\theta_3 = \pi/4$. We see from Eq.(2) that solar neutrino oscillations are maximal:

$$P(\nu_e \to \nu_e)\Big|_s = 1 - \sin^2(\Delta_s R_s/4E) \tag{6}$$

Moreover, we see from Eq.(2) that atmospheric ν_μ's oscillate exclusively into ν_τ's with the unconstrained mixing angle θ_1:

$$P(\nu_\mu \to \nu_\tau)\big|_a = \sin^2 2\theta_1 \, \sin^2 (\Delta_a R_a/4E) P(\nu_\mu \leftrightarrow \nu_e)\big|_a = 0 \qquad P(\nu_e \to \nu_\tau)\big|_a = 0 \tag{7}$$

This implementation is compatible with experiment: It predicts maximal energy-independent solar oscillations and it remains consistent with the solar data if chlorine results are dropped.

The advantage of the model is its economy: no additional chiral fermions are present and so, unlike other extensions[6] of the standard model anomaly cancellation does not lead to any new restrictions, and the number of added parameters is small.

1. P.H. Frampton and S.L. Glashow, Phys. Lett. **B461,** 95 (1999).
2. H. Georgi and S.L. Glashow, `hep-th/9808293`
3. A. Zee, Phys. Lett. **93B,** 389 (1980).
4. C. Jarlskog *et al.*, Phys. Lett. **B449,** 240 (1999);
   M. Matsuda *et al.*, `hep-ph/0005147`.
5. A. Yu. Smirnov and Zhijian Tao, Nucl. Phys. B426 (1994) 415;
   A. Yu. Smirnov and M. Tanimoto, Phys. Rev. D55 (1997) 1665.
6. P.H. Frampton and S.L. Glashow, Phys. Lett. **B190,** 157 (1987); Phys. Rev. Lett. **58,** 2168 (1987). P.H. Frampton, Phys. Rev. Lett. **69,** 2889 (1992).